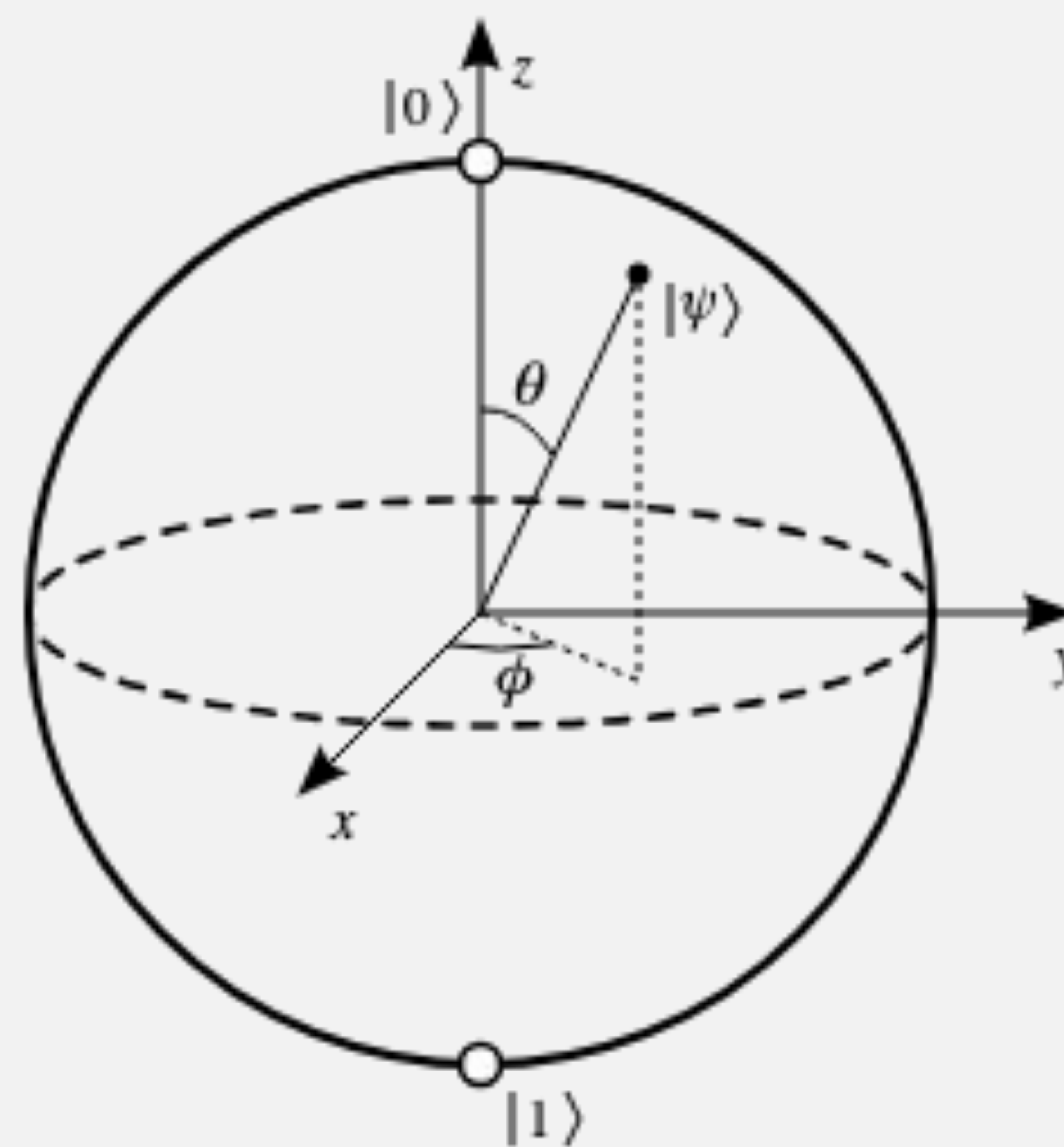


Demo Presentation

Why Quantum Natural Gradients Fail

Project Team:

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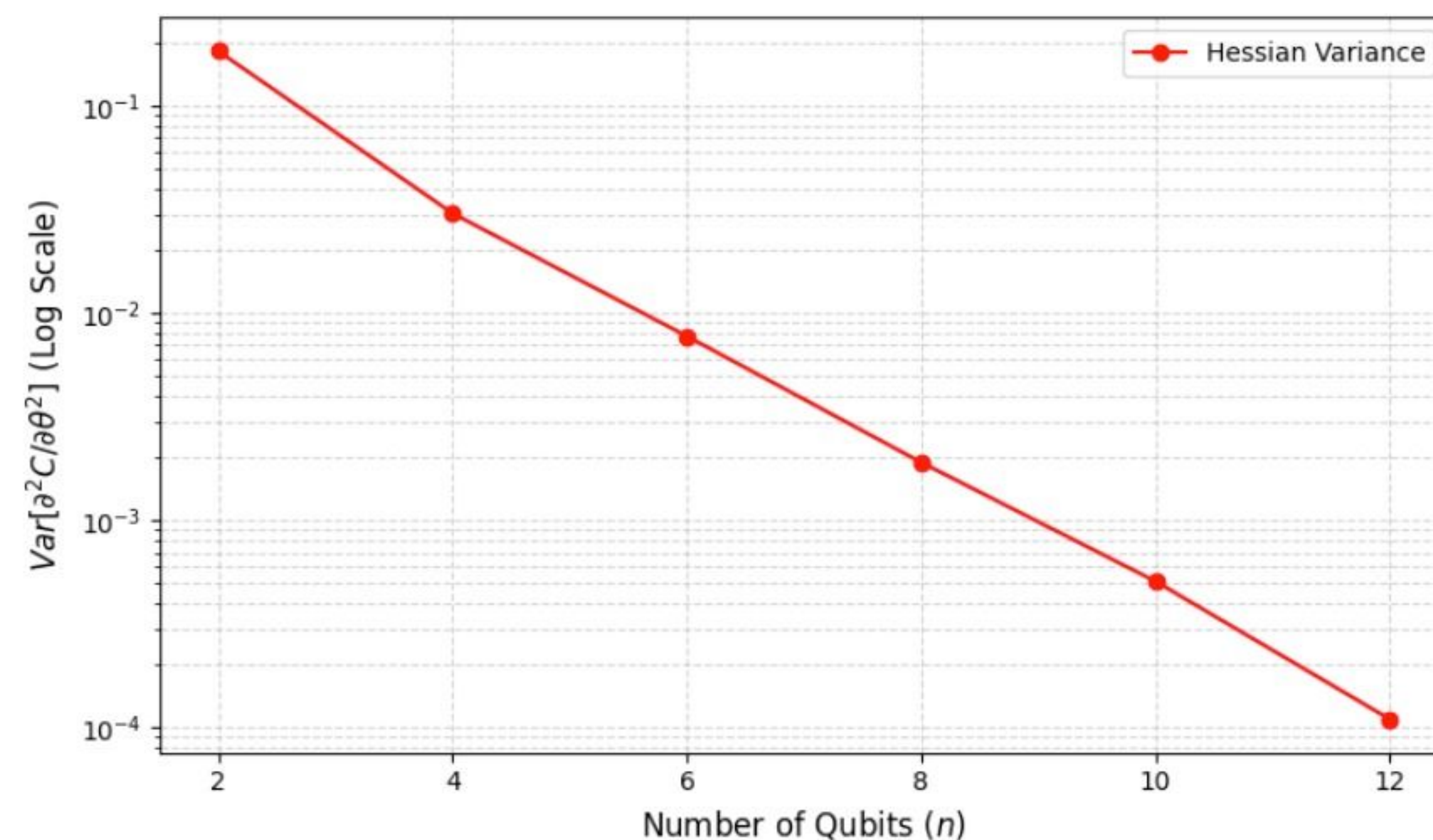
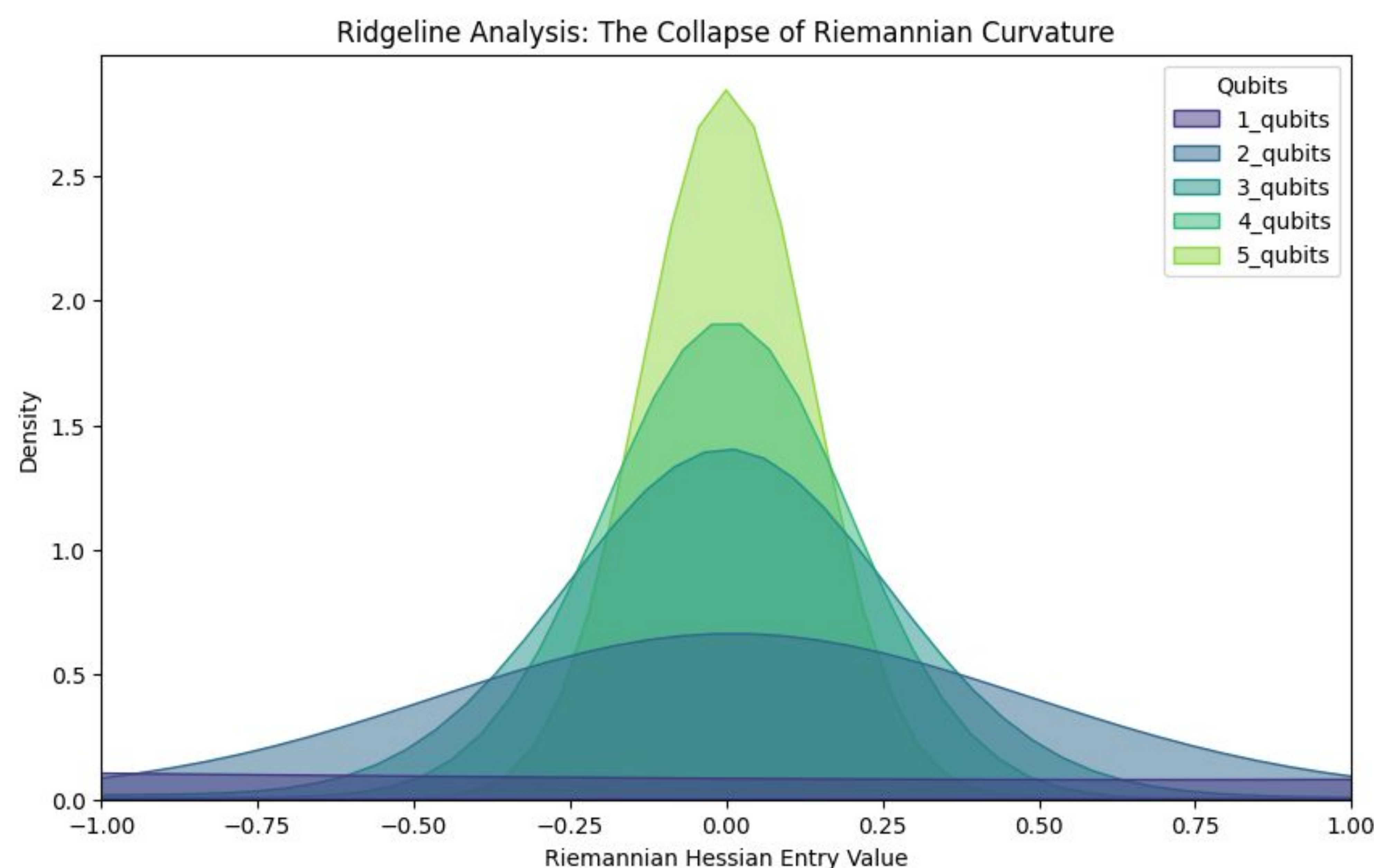


Hessian-Variance

Theorem 4 (Riemannian Barren Plateau). *Let $\mathcal{M}$ be a quantum statistical manifold generated by a deep variational ansatz approximating at least a 2-design. The variance of the Riemannian Hessian elements is exponentially suppressed with respect to the number of qubits n , scaling as:*

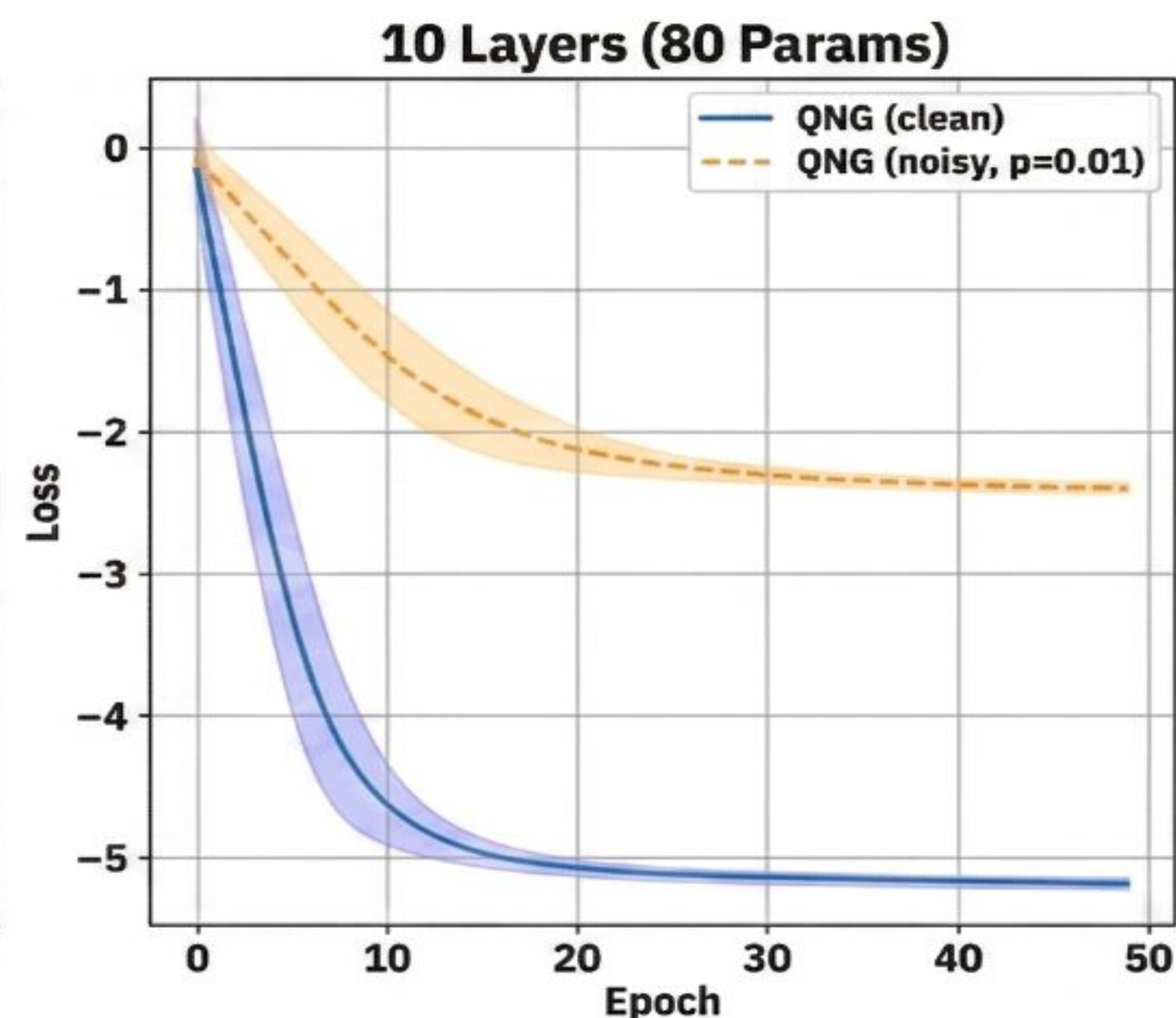
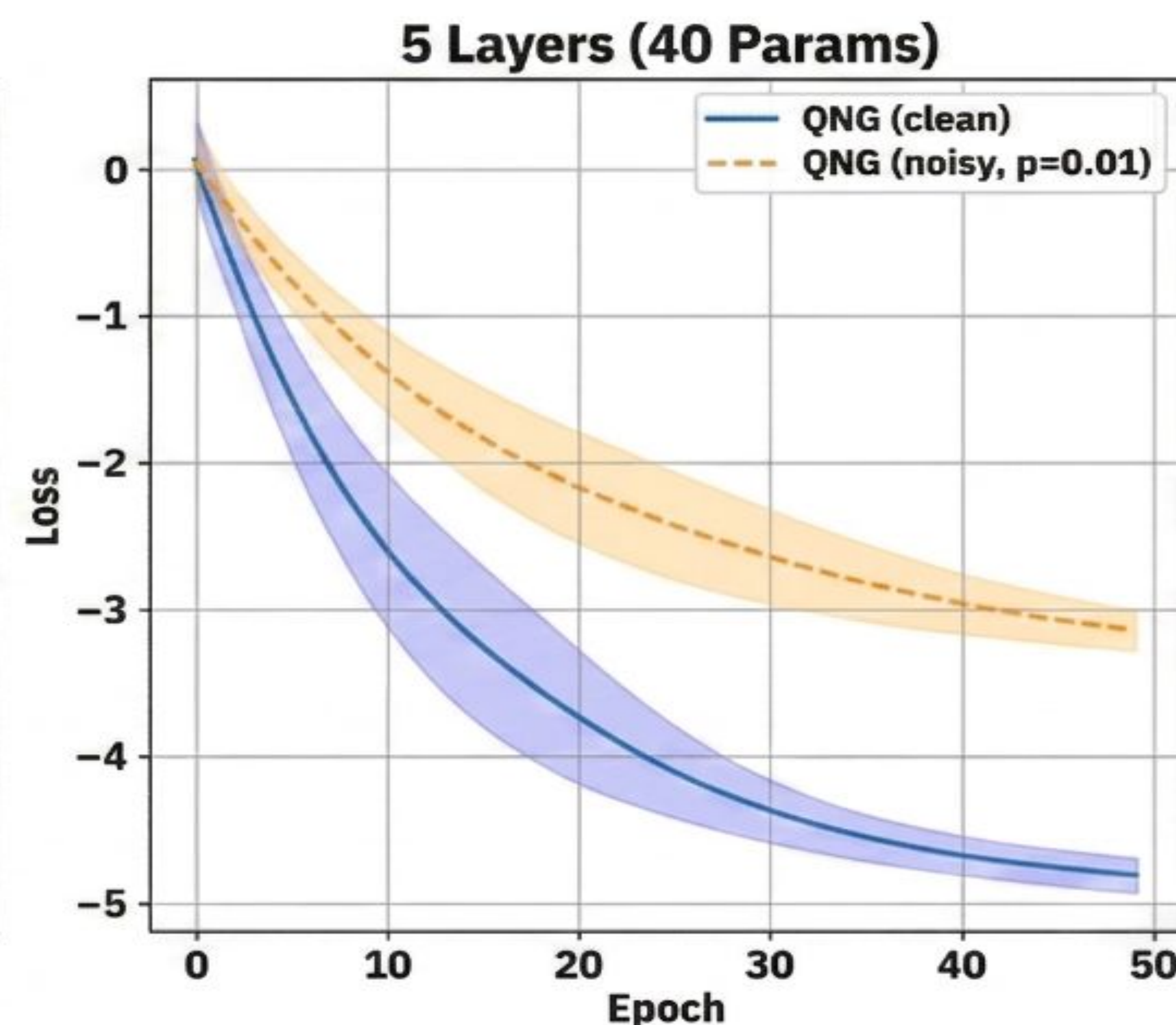
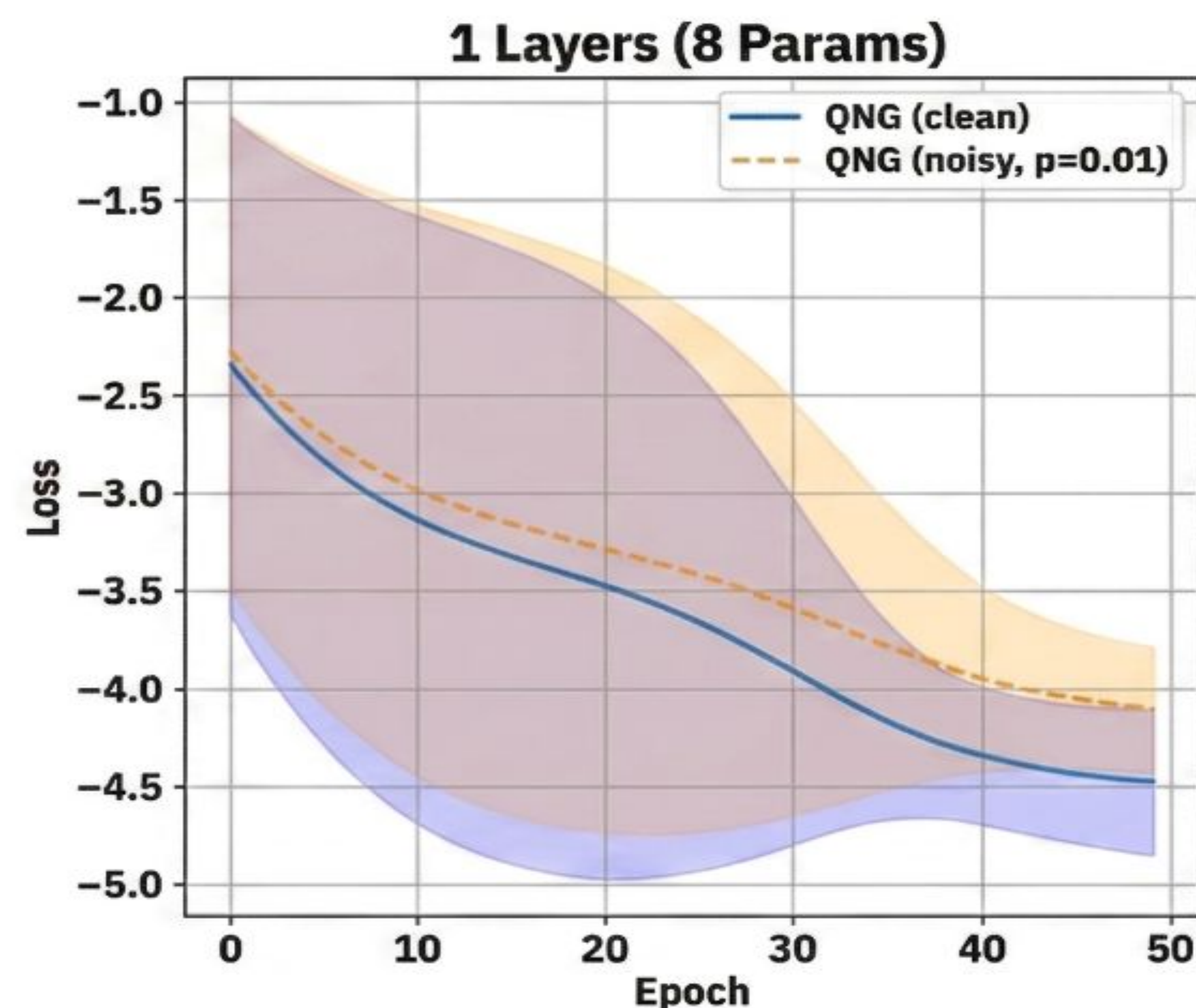
$$\text{Var}[\mathbf{H}_{ij}^R] \in \mathcal{O}\left(\frac{\text{Poly}(P)}{4^n}\right)$$

Riemannian Barren Plateau



QNG Noise

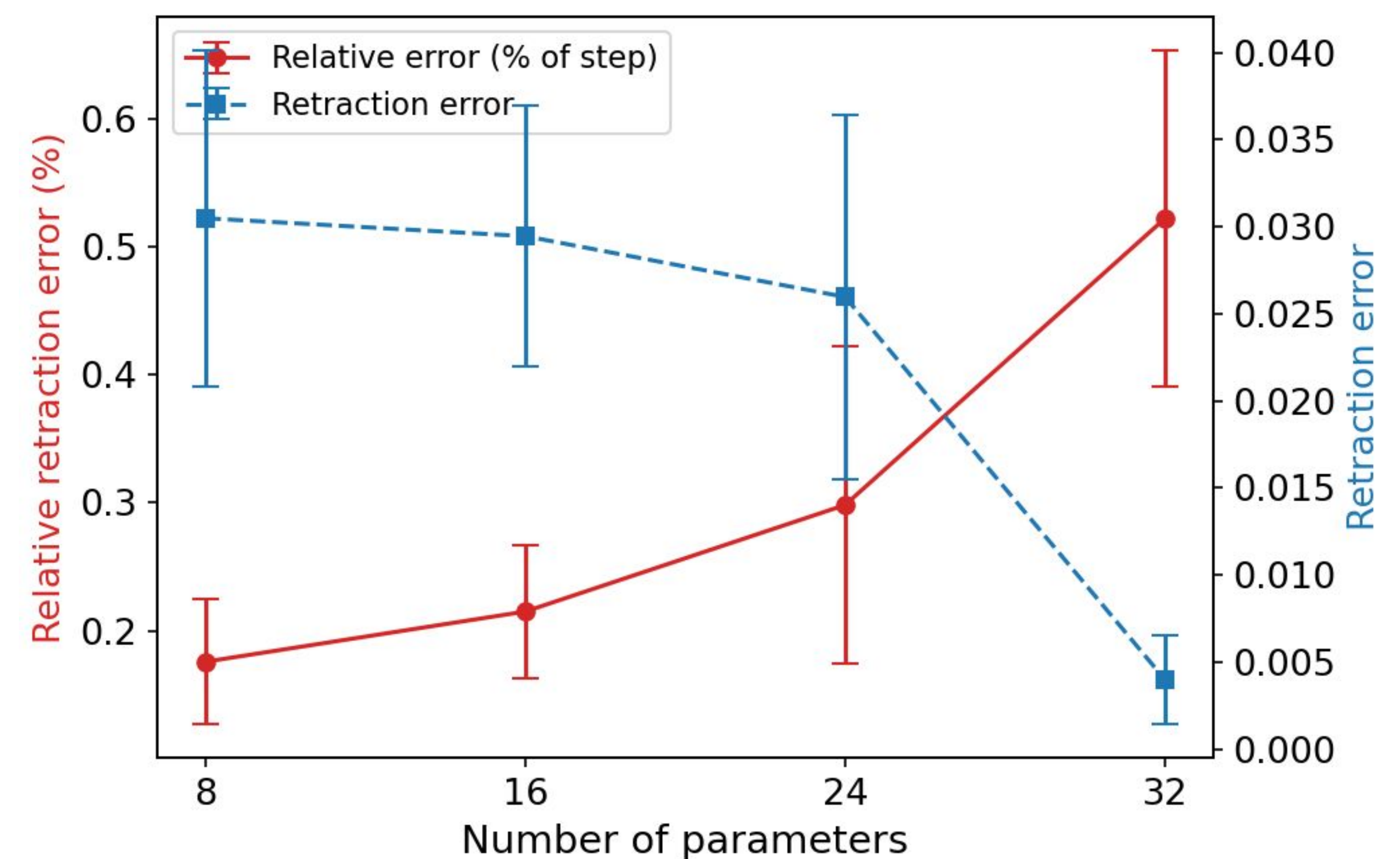
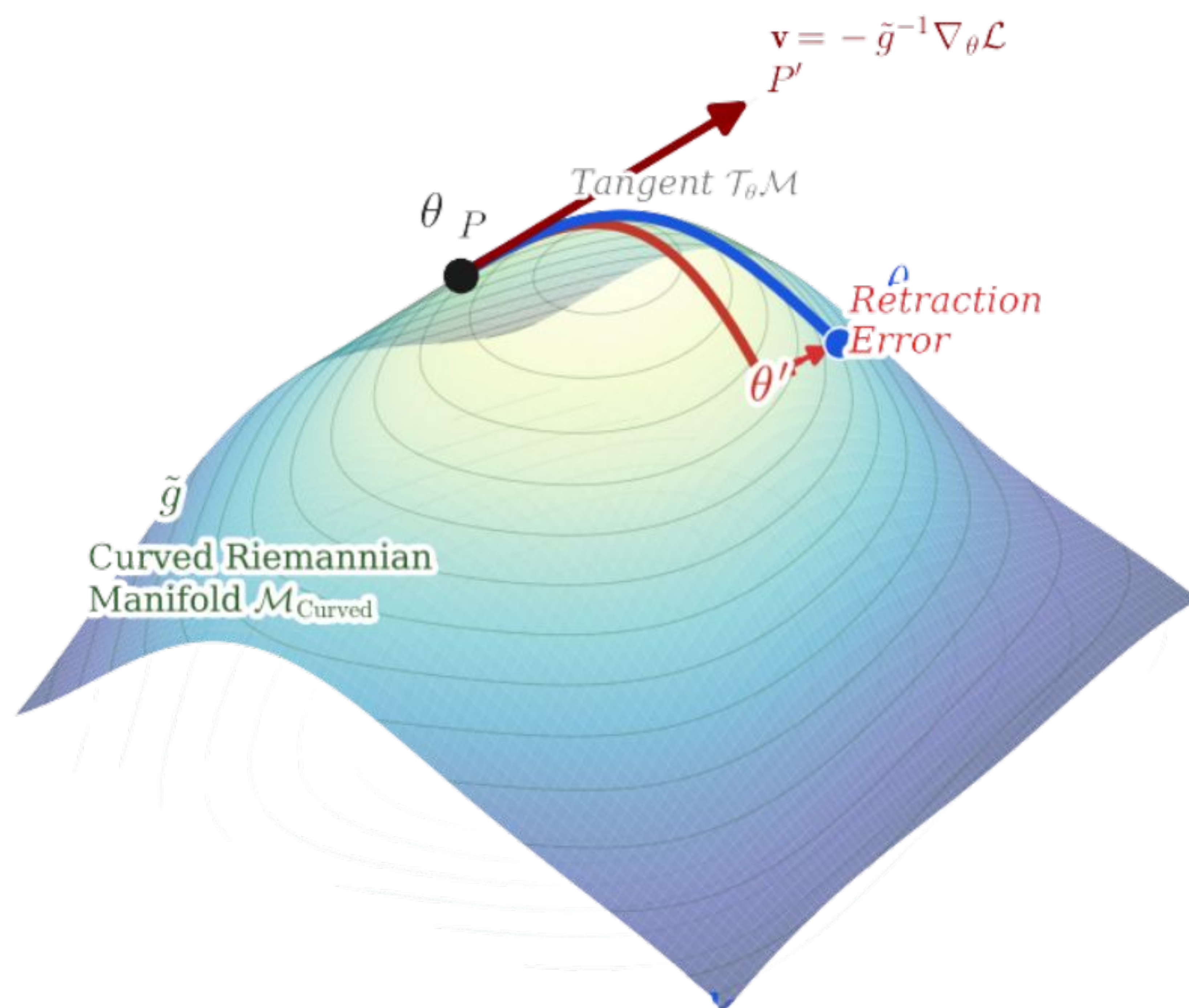
Theorem 3 (Noise-Induced Geodesic Deviation of QNG). *Let $\mathcal{M}$ be the quantum statistical manifold with the regularized Fubini-Study metric $\tilde{g}$, subjected to local depolarizing noise with contraction parameter $q < 1$ and circuit depth L . While the Levi-Civita connection governing the manifold's intrinsic curvature remains invariant to leading order, the contravariant natural gradient vector field $\tilde{\mathbf{v}}$ is exponentially amplified. Consequently, the geodesic deviation of the uncorrected ambient retraction diverges exponentially as $\|\tilde{\mathcal{E}}\|_g \in \mathcal{O}(q^{-2cL})$.*



Retraction Scaling

Lemma 1 (Intrinsic Retraction Error of the Quantum Natural Gradient). *Let $\mathcal{M}$ be the quantum statistical manifold with the regularized Fubini-Study metric $\tilde{g}$. Consider a point $\theta \in \mathcal{M}$ and the natural gradient vector field $\vec{v}$. For a step size $\eta > 0$, let $\mathcal{E}$ denote the geometric error vector between the retraction $\mathbb{R}_\theta(\eta\vec{v})$ and the exponential map $\exp_\theta(\eta\vec{v})$. The intrinsic magnitude of this deviation, evaluated via the induced Riemannian metric, is strictly bounded by:*

$$\|\mathcal{E}^k\|_{\tilde{g}} \leq \sqrt{2 \sum_{k=1}^P \|\mathfrak{g}_k\|_2^2} \left(\frac{18\eta^2 \|\dot{O}\|_2^2 \sqrt{P}}{\epsilon^3} \left(\sum_{l=1}^P \|\mathfrak{g}_l\|_2 \right) \left(\sum_{m=1}^P \|\mathfrak{g}_m\|_2^2 \right)^2 \right) \quad (8)$$

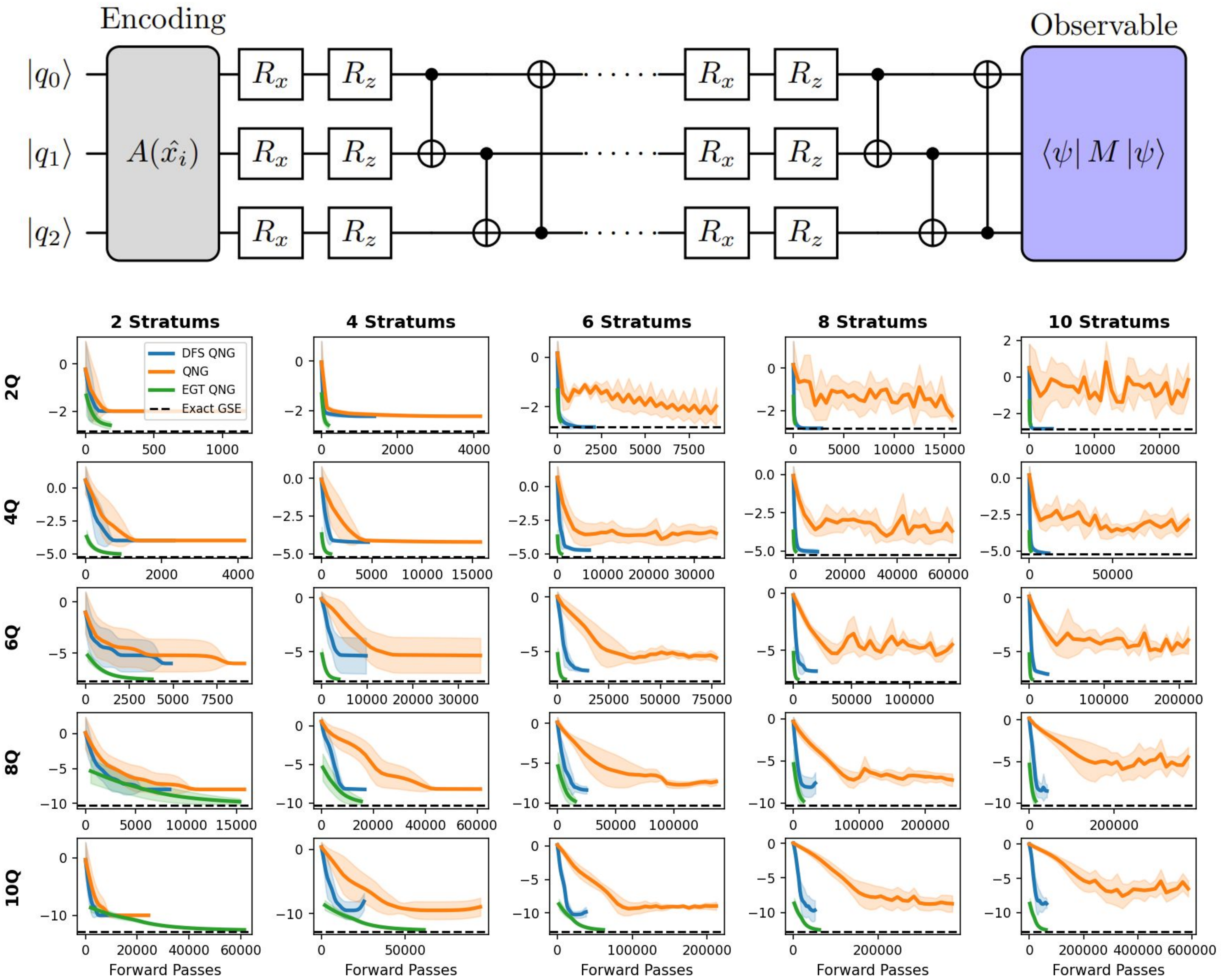


Dually Flat Stratified QNG

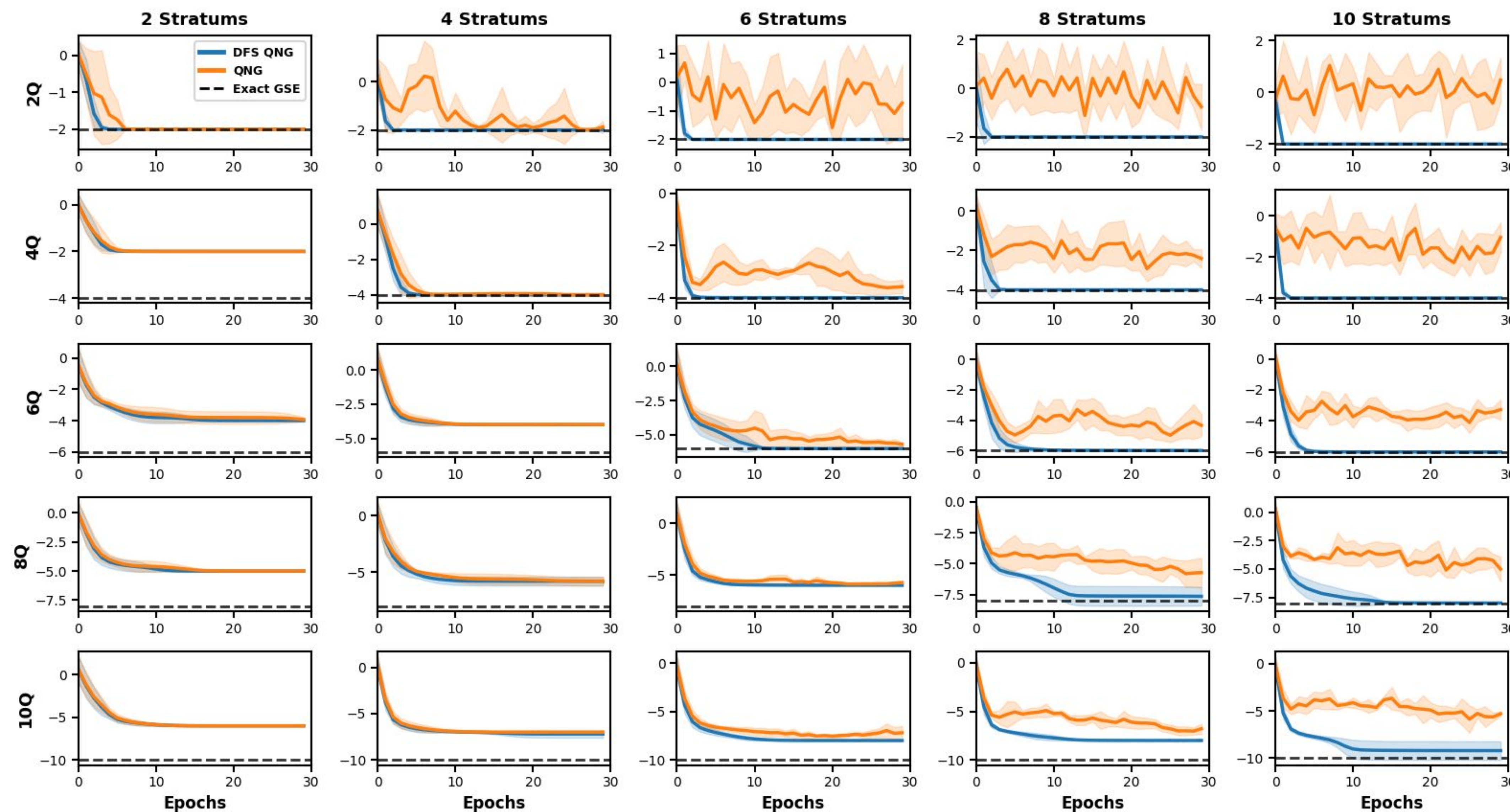
Algorithm 1 Dually-Flat Stratified Geodesic Quantum Natural Gradient (Exact QNG)

Require: Initial parameters $\theta^{(0)} \in \Theta \subseteq \mathbb{R}^P$; Abelian-stratified ansatz $\mathcal{U}(\theta)$ with stratification $f(\mathfrak{g}, \Theta) = \bigsqcup_{\pi=1}^{\mathfrak{S}} \mathcal{S}_{\pi}$ satisfying (16); shift parameter $s \in \mathbb{R}^+$; step size $\eta \in \mathbb{R}^+$; optimization horizon $T \in \mathbb{N}$.

- 1: **for** $t = 0, 1, \dots, T - 1$ **do**
- 2: **for** $\pi = 1, 2, \dots, \mathfrak{S}$ **do**
- 3: **Submanifold Restriction.** Project onto $\mathcal{S}_{\pi}$: extract active coordinates $\theta_{\pi}^{(t)} \leftarrow \Pi_{\pi}(\theta^{(t)})$; freeze complement $\theta_{\setminus \pi}^{(t)} \leftarrow (I - \Pi_{\pi})(\theta^{(t)})$.
- 4: **Directional Derivative Evaluation.**
- 5: **for each** $k \in \Theta_{\pi}$ **do**
- 6: $\mathcal{L}_k^{\pm} \leftarrow \mathcal{L}(\theta^{(t)} \pm s e_k)$
- 7: $[\nabla_{\pi} \mathcal{L}]_k \leftarrow (2 \sin s)^{-1}(\mathcal{L}_k^{+} - \mathcal{L}_k^{-})$
- 8: **end for**
- 9: **Stratum Metric Evaluation.** Prepare $|\psi_{\pi}\rangle = \mathcal{U}_{\pi}(\theta_{\pi}^{(t)}) |\phi_{[1:\pi-1]}\rangle$ and evaluate:
- 10: $\tilde{g}_{\pi\pi} \leftarrow \text{Re}[\langle \partial_i \psi_{\pi} | \partial_j \psi_{\pi} \rangle - \langle \partial_i \psi_{\pi} | \psi_{\pi} \rangle \langle \psi_{\pi} | \partial_j \psi_{\pi} \rangle]_{i,j \in \Theta_{\pi}}$
- 11: **Geodesic Flow Integration.** Since $\Gamma_{ij}^{(e)k}|_{\mathcal{S}_{\pi}} = 0$, the exponential map reduces to coordinate translation:
- 12: $\theta_{\pi}^{(t+1)} \leftarrow \theta_{\pi}^{(t)} - \eta \tilde{g}_{\pi\pi}^{-1} \nabla_{\pi} \mathcal{L}$
- 13: **Chart Recombination.** $\theta^{(t+1)} \leftarrow \theta_{\setminus \pi}^{(t)} \oplus \theta_{\pi}^{(t+1)}$
- 14: **end for**
- 15: **end for**
- 16: **return** $\theta^{(T)}$



QNG Optimizer Comparison



-  Dually Flat QNG
-  Plots
-  QNG-Noise
-  RH-Variance
-  Retraction Scaling

<https://github.com/Damrl-lab/DFS-QNG>